Internal Dose Assessment

10.1 Basic Concepts in Internal Dose Calculations

Internal dose concepts are employed when radioactive material enters the body through any pathway, either in a workplace situation, in the nuclear medicine clinic (where the intakes are quite intentional), from eating or drinking contaminated food and water, and other such situations. In the workplace, the most common modes of intake are:

- Inhalation (the material is breathed in, as a dust, gas, or vapor)
- Ingestion (the material is swallowed in food or liquids)
- Injection (the material is directly introduced into the bloodstream, by a puncture wound, adsorption across the skin barrier, or other means)

These modes are in order of likelihood for an occupational setting. In the nuclear medicine clinic, they are typically in the reverse order: injection is the most common, then infrequently some materials are inhaled as gases (to study lung function) or eaten as radioactively labeled meals (to study, e.g., gastric motility). Radioactive material that enters the body will be very mobile. Most external sources are typically stationary and easy to characterize.

Radioactive material that enters the lung or gastrointestinal (GI) tract, moves within these regions, and then may be absorbed into blood, where it is carried around in the body, deposited in different regions, and then cleared from these regions into excretory pathways at different rates. As one cannot measure internal doses (as people tend to object when you try to place dosimeters into their organs), internal doses are instead calculated through the use of mathematical models. The mobile nature of the radioactive source introduces another complexity to the situation that must be dealt with. This chapter provides an overview of models and methods for internal dose assessment (also called internal dosimetry by some, as explained in the introduction to Chapter 9).

To estimate absorbed dose for all significant tissues, one must determine for each tissue the quantity of energy absorbed per unit mass. This yields the quantity of absorbed dose, if expressed in proper units, and can be extended to calculation of dose equivalent if desired. What quantities are then needed to calculate the two key parameters, energy and mass? To facilitate this analysis,
imagine an object that is uniformly contaminated with radioactive material. Depending on the identity of the radionuclide, particles or rays of characteristic energy and abundance will be given off at a rate dependent upon the amount of activity present. Our object must have some mass. Already we have almost all of the quantities needed for our equation: energy per decay (and number per decay), activity, and mass of the target region. One other factor needed is the fraction of emitted energy that is absorbed within the target. This quantity is most often called the absorbed fraction and is represented by the symbol $\phi$.

For photons (gamma rays and X-rays) some of the emitted energy will escape objects of the size and composition of interest to internal dosimetry (mostly soft tissue organs with diameters of the order of centimeters). For electrons and beta particles, most energy is usually considered to be absorbed, so we usually set the absorbed fraction to 1.0. Electrons, beta particles, and the like are usually grouped into a class of radiations referred to as ‘nonpenetrating emissions’, whereas X and gamma rays are called ‘penetrating radiations’. We can show a generic equation for the absorbed dose rate in our object as

$$D = k \frac{A \sum n_i E_i \phi_i}{m}$$

where:

- $D$ = absorbed dose rate (rad/hr or Gy/sec)
- $A$ = activity ($\mu$Ci or MBq)
- $n_i$ = number of radiations with energy $E_i$ emitted per nuclear transition
- $E_i$ = energy per radiation (MeV)
- $\phi_i$ = fraction of energy emitted that is absorbed in the target
- $m$ = mass of target region (g or kg)
- $k$ = proportionality constant (rad-$\mu$Ci-hr-MeV or Gy-kg/MBq-sec-MeV)

It is extremely important that the proportionality constant be properly calculated and applied. The results of our calculation will be useless unless the units within are consistent and they correctly express the quantity desired. The application of radiation weighting factors to this equation to calculate the dose equivalent rate is a trivial matter; for most of this chapter, we consider only absorbed doses for discussion purposes.

The investigator is not usually interested only in the absorbed dose rate; more likely an estimate of total absorbed dose from an administration is desired. In the above equation the quantity activity (nuclear transitions per unit time) causes the outcome of the equation to have a time-dependence. To calculate cumulative dose, the time integral of the dose equation must be calculated. In most cases, the only term that has a time-dependence is activity, so the integral is just the product of all the factors in the above equation and the integral of the time–activity curve.

Regardless of the shape of the time–activity curve, its integral, however obtained, will have units of the number of nuclear transitions (activity, which is transitions per unit time, multiplied by time; Figure 10.1). Therefore, the equation for cumulative dose would be

$$D = \frac{k \ A \sum n_i E_i \phi_i}{m}$$
Effective Half-Time

10.2 Effective Half-Time

We know that radioactive materials decay according to first order kinetics, that is, a certain fraction of the remaining activity is removed during a specific time interval:

\[
\frac{dN}{dt} = -\lambda N.
\]

The well-known solution to this equation is:

\[
N = N_0 e^{-\lambda t}, \quad A = A_0 e^{-\lambda t}.
\]

In these equations, \( N \) is the number of atoms, \( N_0 \) is the initial number of atoms, \( A \) is the amount of activity, and \( A_0 \) is the initial activity (\( A = \lambda N \)). Many materials are also cleared from the body or certain organs by first-order processes. If we develop an equation for the reduction in the amount of a nonradioactive substance by a first-order system, it would look much like the equations above:

\[
X(t) = X_0 e^{-\lambda_b t}
\]

where:

- \( X(t) \) = the amount of the substance at time \( t \)
- \( X_0 \) = the initial amount of substance
- \( \lambda_b \) = the biological disappearance constant = 0.693/\( T_b \)
- \( T_b \) = the biological halftime for removal

A biological halftime for removal is exactly analogous to a radioactive (or physical) halflife; that is, it is the time in which half of the remaining material is removed.

If we now consider a certain amount of radioactive material in the body that is being cleared from the body by a first-order process, two first-order processes are involved in removing activity from the body: radioactive...
decay and biological disappearance. Because the decay constants are essentially probabilities of removal per unit time, the disappearance constants for the two processes can be added to give an “effective disappearance constant:"

\[ \lambda_e = \lambda_b + \lambda_p, \]

where:

- \( \lambda_e \) = effective disappearance constant
- \( \lambda_p \) = radioactive (physical) decay constant
- \( \lambda_b \) = biological disappearance constant

We can also define an “effective halftime” equal to \( \frac{0.693}{\lambda_e} \), which is the actual time for half of the activity to be removed from the body or organ. The effective halftime is related to the other two half-times by the relationship:

\[ T_e = \frac{T_b \times T_p}{T_b + T_p} \]

For materials that can be described by this type of relationship, the integral of the time–activity curve may be easily evaluated:

\[ \tilde{A} = \int_{0}^{\infty} A(t)dt = \int_{0}^{\infty} (fA_0)e^{-\lambda_e t} dt = \frac{(fA_0)}{\lambda_e} = 1.443 fA_0 T_e, \]

where \( A_0 \) is the administered activity, and \( f \) is the fraction of administered activity in a region at time zero. So, effective halftime is a critical parameter in the determination of cumulated activity and cumulative dose.

Note that the effective half-time for a compound will always be less than or equal to the shorter of either the biological or radiological half-time. As two processes are contributing to the removal of the element, the action of the two together must be faster than that of either acting alone. Note also that to solve the equation for effective half-time, the units for the biological and physical half-times must be the same.

**Examples**

\[
\begin{align*}
T_b &= 7 \text{ days} \quad T_p = 10 \text{ days} \quad T_{eff} = \frac{10 \times 7}{10 + 7} = 4.12 \text{ days} \\
T_b &= 7 \text{ days} \quad T_p = 7 \text{ days} \quad T_{eff} = \frac{7 \times 7}{7 + 7} = 3.5 \text{ days}.
\end{align*}
\]

(Note: This is not a coincidence. Every time that the biological and physical half-times are the same, the effective half-time is exactly half of either value, because the expression is \((x \cdot x)/2x = x/2\).)

\[
\begin{align*}
T_b &= 7 \text{ days} \quad T_p = 100 \text{ days} \quad T_{eff} = \frac{100 \times 7}{100 + 7} = 6.54 \text{ days} \\
T_b &= 7 \text{ days} \quad T_p = 10^6 \text{ days} \quad T_{eff} = \frac{10^6 \times 7}{10^6 + 7} = 7.00 \text{ days}.
\end{align*}
\]
10.3 Dosimetry Systems

So, as one half-time gets very long relative to the other, the effective half-time approaches the shorter of the two.

\[
D = \int D \, dt = \frac{k}{m} \sum_i n_i E_i \phi_i
\]

\[
\int A \, dt = \int f A_0 e^{-\lambda_c t} \, dt = \frac{f A_0}{\lambda_c} (1 - e^{-\lambda_c t})
\]

If we integrate from 0 to \(\infty\), this turns out to be just \(A_0/\lambda_c\). Life is good.

\[
D = \frac{k \int A \, dt \sum_i n_i E_i \phi_i}{m}
\]

\[
D = \frac{k \tilde{A} \sum_i n_i E_i \phi_i}{m}
\]

\[
D = \frac{k \cdot 1.443 \cdot f \, A_0 \, T_c \sum_i n_i E_i \phi_i}{m}
\]

Now consider that we have two objects that are contaminated with radioactive material, and are able to irradiate themselves, each other, and possibly other objects in the system:

\[
D_1 = \frac{k \tilde{A}_1 \sum_i n_i E_i \phi_i (1 \leftarrow 1)}{m_1} + \frac{k \tilde{A}_2 \sum_i n_i E_i \phi_i (1 \leftarrow 2)}{m_1} + \ldots
\]

\[
D_2 = \frac{k \tilde{A}_1 \sum_i n_i E_i \phi_i (2 \leftarrow 1)}{m_2} + \frac{k \tilde{A}_2 \sum_i n_i E_i \phi_i (2 \leftarrow 2)}{m_2} + \ldots
\]

10.3 Dosimetry Systems

The equations above are generic cumulative dose equations. Many authors have developed this equation in one form or another to apply to different situations. Usually many of the factors in the equations are grouped together to simplify calculations, particularly for radionuclides with complex emission spectra. Some of the physical quantities such as absorbed fraction and mass can also be combined into single values. However these quantities may be
grouped, hidden, or otherwise moved around in different systems, all of them incorporate the concepts from these equations, and all are based on the same principles. Given the same input data and assumptions, the same results will be obtained. Sometimes, the apparent differences between the systems and their complicated-appearing equations may confuse and intimidate the user who may be frustrated in trying to make any two of them agree for a given problem. Careful investigation to discern these grouped factors can help to resolve apparent differences. Let us try to understand each of the systems and see how they are equivalent.

10.3.1 Marinelli–Quimby Method

Publications by Marinelli et al. and Quimby and Feitelberg\(^1,2\) gave the dose from a beta emitter that decays completely in a tissue as

\[ D_\beta = 73.8 \cdot C \cdot E_\beta \cdot T, \]

where \( D_\beta \) is the dose in rad, \( C \) is the concentration of the nuclide in microcuries per gram, \( E_\beta \) is the mean energy emitted per decay of the nuclide, and \( T \) is the half-life of the nuclide in the tissue. We have seen that the cumulated activity is given as 1.443 times the half-life times the initial activity in the tissue. The other terms in the equation are:

\[ k = \left( \frac{73.8}{1.443} \right) = 51.1; \]

\( C \) is activity per mass; and for beta emitters, we assume that \( \phi \) is 1.0. For gamma emitters, values of \( \phi \) were estimated from the geometrical factors of Hine and Brownell\(^3\) for spheres and cylinders of fixed sizes. Dose rates were based on expressions for dose near a point-source gamma emitter integrated over the source volume:

\[ D_\gamma = 10^{-3} \cdot \Gamma \cdot C \cdot \int_v \frac{e^{-\mu r}}{r^2} \cdot dV \cdot \frac{\text{rad}}{h} \]

It is difficult to see how this equation fits the form of our general equation, but it does. The factor \( C \) is still the activity per unit mass. The specific gamma rate constant \( \Gamma \) essentially gives the exposure rate per disintegration into an infinite medium from a point-source (equivalent to \( k \times \Sigma n_i \times E_i \) in our generic equation). Finally, the factor \( \left[ \frac{\exp(-\mu r)}{h^2} \cdot dV \right] \) acts as an absorbed fraction (\( \mu \) is an absorption coefficient and \( 1/r^2 \) is essentially a geometrical absorbed fraction). The integral in this expression can be obtained analytically only for simple geometries. Solutions for several standard objects (spheres, cylinders, etc.) were provided in the geometrical factors in Hine and Brownell’s text.

10.3.2 International Commission on Radiological Protection

The ICRP has developed two comprehensive internal dosimetry systems intended for use in occupational settings (mainly the nuclear fuel cycle). ICRP publication II\(^4\) became part of the basis for the first set of complete radiation protection regulations in this country (Code of Federal Regulations (CFR), Title 10, Chapter 20, or 10 CFR 20). These regulations were only replaced (completely) in 1994 when a revision of 10 CFR 20 incorporated the new procedures and results of the ICRP 30 series.\(^5\) Even these two systems, published by the same body, appear on the surface to be completely different.
We have already noted, however, that they are completely identical in concept and differ only in certain internal assumptions. Both of these systems, dealing with occupational exposures, were used to calculate dose equivalent instead of just absorbed dose.

In the ICRP II system, the dose equivalent rate is given by

$$H = \frac{51.2 \ A\xi}{m}.$$  

This looks somewhat like our original equation, converted to dose equivalent, but a lot seems to be missing. The missing components are included in the factor $\xi$:

$$\xi = \sum n_i \ E_i \ \phi_i \ Q_i.$$

The factor 51.2 is $k$, which puts the equation into units of rem per day, for activity in microcuries, mass in grams, and energy in megaelectron volts (and note that the ICRP included a quality factor, $Q$, to express the results in equivalent dose). The ICRP developed a system of limitation of concentrations in air and water for employees from this equation and assumptions about the kinetic behavior of radionuclides in the body. These were the well-known Maximum Permissible Concentrations (MPCs). Employees could be exposed to these concentrations on a continuous basis and not receive an annual dose rate to the so-called critical organ that would exceed established limits.

In the ICRP 30 system, the cumulative dose equivalent is given by

$$H_{50,T} = 1.6 \times 10^{-10} \sum_S U_S \ SEE(T \leftarrow S).$$

In this equation, $T$ represents a target region and $S$ represents a source region.

This equation looks altogether new; nothing much is similar to any of the previous equations. This is simply, however, the same old equation wearing a new disguise. The factor $SEE$ is merely

$$SEE = \sum_i \frac{n_i \ E_i \ \phi_i(T \leftarrow S) \ Q_i}{m_T}.$$

The factor $U_s$ is another symbol for cumulated activity, and the factor $1.6 \times 10^{-10}$ is $k$. Note that the symbol $Q$ (quality factor) is shown here instead of the current notation $w_R$ (radiation weighting factor), as this is how it appeared in ICRP 30. In this system (based on the Système International unit system), this value of $k$ produces cumulative dose equivalents in sievert, from activity in becquerels, mass in grams, energy in megaelectron volts, and appropriate quality factors. As in ICRP II, this equation was used to develop a system of dose limitation for workers, but unlike the ICRP II system, limits are placed on activity intake during a year, which would prevent cumulative doses (not continuous dose rates) from exceeding established limits. These quantities of activity were called Annual Limits on Intake (ALIs); derived air concentrations, which are directly analogous to MPCs for air, are calculated from ALIs.

The real innovation in the ICRP 30 system is the so-called Effective Dose Equivalent ($H_e$ or EDE). As we defined in Chapter 7, certain organs or organ systems were assigned dimensionless weighting factors that are a function of their assumed relative radiosensitivity for expressing fatal cancers or genetic
defects. The assumed radiosensitivities were derived from the observed rates of expression of these effects in various populations exposed to radiation. Multiplying an organ’s dose equivalent by its assigned weighting factor gives a weighted dose equivalent. The sum of weighted dose equivalents for a given exposure to radiation is the effective dose equivalent. It is the dose equivalent that, if received uniformly by the whole body, would result in the same total risk as that actually incurred by a nonuniform irradiation. It is entirely different from the dose equivalent to the whole body that is calculated using values of SEE for the total body. Whole-body doses are often meaningless in internal dose situations because nonuniform and localized energy deposition is averaged over the mass of the whole body (70 kg).

One real difference that exists between doses calculated with the ICRP II system and the ICRP 30 (and MIRD) system is that the authors of ICRP II used a very simplistic phantom to estimate their absorbed fractions. All body organs and the whole-body were represented as spheres of uniform composition. Furthermore, organs could only irradiate themselves, not other organs. So, although contributions from all emissions were considered, an organ could only receive a dose if it contained activity, and the absorbed fractions for photons were different from those calculated from the more advanced phantoms used by ICRP 30 and MIRD (described next).

10.3.3 Medical Internal Radiation Dose (MIRD) System

The equation for absorbed dose in the MIRD system \(^6\) is deceptively simple:

\[
D_r = \sum_h \tilde{A}_h S(r_k \leftarrow r_h)
\]

In this equation, \(r_k\) represents a target region and \(r_h\) represents a source region. No one is fooled by now, of course. The cumulated activity is there; all other terms must be lumped in the factor \(S\), and so they are:

\[
S(r_k \leftarrow r_h) = \frac{k \sum_i n_i E_i \phi_i(r_k \leftarrow r_h)}{m_{r_k}}
\]

In the MIRD equations, the factor \(k\) is 2.13, which gives doses in rad, from activity in microcuries, mass in grams, and energy in MeV. The MIRD system was developed primarily for use in estimating radiation doses received by patients from administered radiopharmaceuticals; it was not intended to be applied to a system of dose limitation for workers. In the MIRD system, one may sometimes also see the use of the term \(\Delta_i\). The factor \(\Delta_i = k \times n_i \times E_i\) for a given radionuclide emission, and the equations may be represented as

\[
S(r_k \leftarrow r_h) = \frac{\sum_i \Delta_i \phi_i(r_k \leftarrow r_h)}{m_{r_k}}
\]

\[
D_r = \sum_h \tilde{A}_h S(r_k \leftarrow r_h) = \sum_h \tilde{A}_h \frac{\sum_i \Delta_i \phi(r_k \leftarrow r_h)}{m_{r_k}}
\]
10.3.4 RADAR

In the early 21st century, an electronic resource was established on the Internet to provide rapid worldwide dissemination of important dose quantities and data. The RAdiation Dose Assessment Resource established a Web site at www.doseinfo-radar.com, and provided a number of publications on the data and methods used in the system. The RADAR system has about the simplest manifestation of the dose equation:

\[ D = N \times DF \]

where \( N \) is the number of disintegrations that occur in a source organ, and \( DF \) is:

\[ DF = \sum_{i} n_{i} E_{i} \phi_{i} \]

The \( DF \) is mathematically the same as an “\( S \) value” as defined in the MIRD system. The number of disintegrations is the integral of a time–activity curve for a source region. RADAR members produced compendia of decay data, dose conversion factors, and catalogued standardized dose models for radiation workers and nuclear medicine patients, among other resources. They also produced the widely used OLINDA/EXM personal computer software code, which used the equations shown here and the input data from the RADAR site.

10.4 Internal Dose Calculations for Radiation Workers

Radiation workers are a very important group for which internal dose calculations are performed. Loose radioactive contamination in the workplace may be taken into the body, with the deposition of energy in many body tissues. In the majority of workplace situations, the design goal is to avoid all intakes of radioactive materials. Such intakes generally represent minor or major failures of containment and monitoring systems, accidental situations, or other avoidable scenarios. In some industries, however, such as uranium and thorium mining and milling, airborne radioactive dust is everywhere, and a certain low level of intake of radioactive material is to be expected. In the first case, monitoring for radioactive material will generally show negative results most of the time (i.e., results that are less than the detection system’s MDA; see Chapter 8). In the second case, a certain low level of body retention or excretion is to be expected (above the natural levels of such elements in the environment (this difference may be difficult to characterize) and monitoring is performed to ensure that the levels of intake are in the acceptable range.

Modes of Intake

As noted in the beginning of the chapter, the most common mode of intake in occupational settings is inhalation, followed by ingestion and then injection. Airborne radioactive contaminants may be in the form of aerosols, dusts, or vapors. In particular, iodine (e.g., \(^{131}\text{I}, ^{125}\text{I}\)) is volatile and working with liquid solutions always has the hazard of inhalation of volatilized activity. Mobile \(^{3}\text{H}\) will most often exist as \(^{3}\text{H}_{2}\text{O}\) vapor in any humid environments. Ingestion is rare, and represents perhaps an accidental intake (contaminated hands touching
the mouth directly, handling food or other substances placed in the mouth), material splashed in the face, or material deposited on the throat when mouth breathing.

Injection incidents are quite rare and almost always accidental. People working with syringes to inject radioactive material always run the risk of a puncture incident, even through protective gloves are worn. Handling glass beakers or other containers of liquid radioactive materials can lead to occasional breakage, with sharp pieces of glass cutting through protective gloves or clothing and cutting the skin. Jagged pieces of radioactive metal, for example, depleted uranium being machined for particular purposes, may also cause direct injection of radioactive material into the bloodstream. Most situations, and thus most mathematical models for treating intakes, however, assume that the intakes were from inhalation or ingestion. Thus much work has been done in developing kinetic models to simulate the movement of radioactive material in these two important organ systems. Note that the organs involved in the deposition of radioactive material and its potential transfer to the systemic circulation and other body organs (respiratory tract airways, lungs, and pulmonary lymph nodes in the case of inhalation and the stomach and intestines in the case of ingestion) are also organs for which we wish to know the radiation dose. A general systemic model may be shown as in Figure 10.2.

**Kinetic Models for the GI Tract**

The dosimetric model for the gastrointestinal tract (GI tract) given in ICRP 30 (Figure 10.3) is a very simple, straight-through, four-compartment model. The four sections are stomach, small intestine, upper large intestine, and lower large intestine, sometimes abbreviated ST, SI, ULI, and LLI, respectively. Ingestion is a common means of intake of radioactive material, either through swallowing of material somehow introduced into the mouth or through transfer of material from the various regions of the lung system to the throat and subsequent swallowing. The sections of the GI tract are treated as separate target tissues according to the recommendations in the ICRP 30 dosimetric system. They are not assigned any specific weighting factors, however, and the weighting factors recommended for the “remainder” are assumed to apply to any significant committed dose equivalents.
Translocation from one compartment to the next is assumed to be governed by first-order kinetics (exponential removal). The only way for material to leave the system is through excretion from the LLI or absorption from the SI into the transfer compartment. The removal to the transfer compartment is also assumed to be a first-order process. Most often quoted in the literature is the fraction of stable material reaching the body fluids following ingestion, $f_1$. The transfer coefficient from the SI to the transfer compartment is called $\lambda_B$ and is defined numerically as

$$\lambda_B = \frac{f_1 \lambda_{SI}}{1 - f_1}.$$

Transfer rates are assumed to be fixed for the various compartments regardless of the chemical form of the material. The only parameter affected by the chemical form is the value $f_1$ (and therefore $\lambda_B$). Values of $f_1$ may be given for several chemical forms of the elements. When $f_1$ is very small, most of the activity will pass through the GI tract and be excreted in the feces. Thus, the principal radiation doses of concern will be to the segments of the GI tract itself. When $f_1$ is large, most of the activity ingested will be transferred to the systemic circulation and little will pass through the GI tract beyond the small intestine. When $f_1$ is 1.0 (i.e., 100%) the activity is assumed to pass into the systemic circulation, the transfer is assumed to occur very quickly, and the GI tract organs are not involved at all. Essentially the model assumes that the material passes directly from the stomach to the circulation, as if the material were actually just injected at time zero into the blood.

With intermediate values of $f_1$, both the systemic organs and the GI tract organs may receive significant radiation doses. Some representative $f_1$ values are shown in Table 10.1. (Note: In many cases, more than one chemical class of an element may be known and defined in the ICRP models, so
Table 10.1 Selected value of $f_1$.

<table>
<thead>
<tr>
<th>Element</th>
<th>$f_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>1.0</td>
</tr>
<tr>
<td>Cesium</td>
<td>1.0</td>
</tr>
<tr>
<td>Iodine</td>
<td>1.0</td>
</tr>
<tr>
<td>Cobalt</td>
<td>0.3</td>
</tr>
<tr>
<td>Iron</td>
<td>0.1</td>
</tr>
<tr>
<td>Gallium</td>
<td>0.001</td>
</tr>
<tr>
<td>Uranium</td>
<td>0.05</td>
</tr>
<tr>
<td>Plutonium</td>
<td>0.001</td>
</tr>
</tbody>
</table>

there may be more than one $f_1$ value for the different inhalation classes, as defined below. These are typical values for often encountered classes of these elements.

If you use the values of $\lambda$ given as $s^{-1}$, the expression for the number of transformations occurring in the stomach after intake of 1 Bq of activity is:

$$\tilde{A}_{ST} = \frac{1}{\lambda_{ST} + \lambda_R},$$

where:

$\lambda_{ST}$ = the rate constant for loss of stable material from the stomach to small intestine (24 day$^{-1}$ = 1 hr$^{-1}$ = 0.00028 s$^{-1}$)

$\lambda_R$ = the radioactive decay constant for the radionuclide (s$^{-1}$)

The expression for the number of transformations in the small intestine after ingestion of 1 Bq of activity is:

$$\tilde{A}_{SI} = \frac{\lambda_{SI}}{(\lambda_{ST} + \lambda_R)(\lambda_{SI} + \lambda_B + \lambda_R)},$$

where:

$\lambda_{SI}$ = rate constant for loss of stable material from small intestine to upper large intestine (6.0 day$^{-1}$ = 0.25 hr$^{-1}$ = 6.9 $\times$ 10$^{-5}$ s$^{-1}$)

$\lambda_B$ = rate constant for loss of stable material from small intestine to transfer compartment, as defined above (s$^{-1}$)

The expression for the number of transformations in the upper large intestine after ingestion of 1 Bq is:

$$\tilde{A}_{ULI} = \frac{\lambda_{ST} \lambda_{SI}}{(\lambda_{ST} + \lambda_R)(\lambda_{SI} + \lambda_B + \lambda_R)(\lambda_{ULI} + \lambda_R)},$$

where $\lambda_{ULI}$ = rate constant for loss of stable material from upper large intestine to large intestine (1.8 day$^{-1}$ = 0.075 hr$^{-1}$ = 2.1 $\times$ 10$^{-5}$ s$^{-1}$).

The expression for the number of transformations in the lower large intestine after ingestion of 1 Bq is:

$$\tilde{A}_{LLI} = \frac{\lambda_{ST} \lambda_{SI} \lambda_{ULI}}{(\lambda_{ST} + \lambda_R)(\lambda_{SI} + \lambda_B + \lambda_R)(\lambda_{ULI} + \lambda_R)(\lambda_{LLI} + \lambda_R)}.$$
where $\lambda_{LLI}$ = rate constant for loss of stable material from lower large intestine to the large intestine (1.0 day$^{-1}$ = 0.0417 hr$^{-1}$ = $1.16 \times 10^{-5}$ s$^{-1}$).

If you watched closely as the expressions developed, you might have noticed that a pattern developed through which the number of transformations in one compartment could be predicted by the number in the previous compartment:

$$\tilde{A}_i = \frac{\tilde{A}_{i-1} \lambda_{i-1}}{\lambda_i + \lambda_R}.$$  

Radioactive Progeny

The expressions for production of radioactive daughters within the tract are equally straightforward, but much more complicated. The expressions are shown below:

$$\tilde{\tilde{A}}_{ST-\text{progeny}} = \frac{\tilde{A}_{ST-\text{parent}} \lambda_R \tilde{\tilde{A}}_{ST-\text{progeny}}}{\lambda_{ST} + \lambda_R \tilde{\tilde{A}}_{ST-\text{progeny}}}$$

$$\tilde{\tilde{A}}_{SI-\text{progeny}} = \frac{\tilde{A}_{SI-\text{parent}} \lambda_R \tilde{\tilde{A}}_{SI-\text{progeny}} \lambda_{ST}}{\left(\lambda_{SI} + \lambda_R \tilde{\tilde{A}}_{SI-\text{progeny}}\right)\left(\lambda_{ST} + \lambda_R \tilde{\tilde{A}}_{ST-\text{progeny}}\right)}$$

$$\tilde{\tilde{A}}_{ULI-\text{progeny}} = \frac{\tilde{A}_{ULI-\text{parent}} \lambda_R \tilde{\tilde{A}}_{ULI-\text{progeny}} \lambda_{SI} \lambda_{ULI}}{\left(\lambda_{SI} + \lambda_R \tilde{\tilde{A}}_{SI-\text{progeny}}\right)\left(\lambda_{ULI} + \lambda_R \tilde{\tilde{A}}_{ULI-\text{progeny}}\right)}$$

$$\tilde{\tilde{A}}_{LLI-\text{progeny}} = \frac{\tilde{A}_{ST-\text{parent}} \lambda_R \tilde{\tilde{A}}_{ST-\text{progeny}} \lambda_{ST} \lambda_{SI} \lambda_{ULI}}{\left(\lambda_{ST} + \lambda_R \tilde{\tilde{A}}_{ST-\text{progeny}}\right)\left(\lambda_{SI} + \lambda_R \tilde{\tilde{A}}_{SI-\text{progeny}}\right)\left(\lambda_{ULI} + \lambda_R \tilde{\tilde{A}}_{ULI-\text{progeny}}\right)}$$

Dose Factors—Absorbed Fractions

Absorbed fractions for photons are calculated by Monte Carlo methods using the Fisher–Snyder phantom, as described shortly. For beta particles, the assumption is made that the dose rate to the surface of the segment wall is half that in the contents, which is calculated by the usual theoretical expression. For alpha particles and fission fragments, this value of one-half is further modified to account for selfabsorption within the contents by multiplying by 0.01. Recoil atoms are assumed not to irradiate the GI tract walls.
Kinetic Model for the Lungs

The lungs are much more complicated than the GI tract to model. Unlike the GI tract model, which has not changed since its creation in 1966, the lung model has been continually evolving to more and more complex forms. In its earliest form, given in ICRP II (1960), the model had only two major compartments, the “upper respiratory tract” and the “lower respiratory tract” (Figure 10.4), with half-lives in the lower respiratory tract of < 1 day for “soluble” materials, 120 days for “insoluble” materials, and 365 days and 1460 days for plutonium and thorium.

The ICRP 30 lung model had eight compartments (ten if you include the lymph node compartment; Figure 10.5), with varying clearance half-times which depended on the type of aerosol assumed to be inhaled (class D, W, or Y, corresponding to clearance times of the order of days, weeks, or years, relating to material that had varying degrees of biological mobility). Material deposited in the upper or lower respiratory tract either moved into the bloodstream or into the GI tract (via mucociliary transport into the esophagus). Although tedious, this model could be solved by hand or with a small computer program or spreadsheet. The model structure is shown in the diagram in Figure 10.5. The deposition model was based on particle size.

The figure shows the deposition model, which gives the deposition probability as a function of a parameter known as the Aerosol Median Aerodynamic Diameter (AMAD). Aerosols encountered in the workplace are often not of uniform diameter, and the size of the particles may cover a large range of values (polydisperse aerosol). Such aerosols will typically have lognormal distributions of particle sizes, and the best parameter to characterize the distribution is the median aerodynamic diameter. This diameter may be characterized according to the particle mass, activity, or other criteria; if we use activity, we define the AMAD. Figure 10.6, from ICRP 30, shows that from particles between 0.2 and 10 µm, the deposition in the tracheobronchial region (TB) is about the same, about 0.08 (or 8% of the inhaled particles), deposition increases dramatically in the nasopharyngeal (NP) region with increasing particle size but decreases in the pulmonary (P), or deep lung, region with increasing particle size.
Figure 10.5 The ICRP 30 lung model.

Figure 10.6 Aerosol deposition model in ICRP 30 lung model.
Table 10.2 Fractions and clearance half-times (d) for the ICRP 30 lung model.

<table>
<thead>
<tr>
<th>Region</th>
<th>Class D</th>
<th></th>
<th>Class W</th>
<th></th>
<th>Class Y</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fraction</td>
<td>$T_{1/2}$</td>
<td>Fraction</td>
<td>$T_{1/2}$</td>
<td>Fraction</td>
<td>$T_{1/2}$</td>
</tr>
<tr>
<td>NP</td>
<td>a 0.5</td>
<td>0.01</td>
<td>0.1</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>b 0.5</td>
<td>0.01</td>
<td>0.9</td>
<td>0.40</td>
<td>0.99</td>
<td>0.40</td>
</tr>
<tr>
<td>TB</td>
<td>c 0.95</td>
<td>0.01</td>
<td>0.5</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>d 0.05</td>
<td>0.2</td>
<td>0.5</td>
<td>0.20</td>
<td>0.99</td>
<td>0.20</td>
</tr>
<tr>
<td>P</td>
<td>e 0.8</td>
<td>0.5</td>
<td>0.15</td>
<td>50</td>
<td>0.05</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>f n/a</td>
<td>n/a</td>
<td>0.4</td>
<td>1.0</td>
<td>0.4</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>g n/a</td>
<td>n/a</td>
<td>0.4</td>
<td>50</td>
<td>0.4</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>h 0.2</td>
<td>0.5</td>
<td>0.05</td>
<td>50</td>
<td>0.15</td>
<td>500</td>
</tr>
<tr>
<td>L</td>
<td>i 1.0</td>
<td>0.5</td>
<td>1.0</td>
<td>50</td>
<td>0.9</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>j n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>0.1</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>

particle size. Thus, larger particles tend to deposit in the upper airways and finer particles in the lower airways.

Once deposition has occurred, the model predicted the clearance from each of the compartments above, using single exponential models. Table 10.2 summarizes the model.

Class D materials are cleared with half-lives of the order of days or shorter, and class W and Y materials are cleared more slowly. For class Y material, there is a fraction of material that ends up in the pulmonary lymph nodes with infinitely long retention (i.e., $T_b = \infty$, $T_{eff} = T_{physical}$). So it would be quite tedious, but one could calculate the number of disintegrations in the lung by adding up all of the components of the deposition and retention, calculating the individual $\lambda$ values, and summing the results.

The ICRP 66 lung model\textsuperscript{11} has 14 compartments, treats not only aerosols but also gases and vapors, and material is translocated using a number of mechanical and biochemical processes in a number of directions. The model can only be solved and used with a fairly complex computer program. The deposition model is more complex (this plot shows the activity median aerodynamic and activity median thermodynamic diameters, in different regions) (Figure 10.7). The retention model has more compartments (Figure 10.8), as noted above,
but also has far more highly complex translocation kinetics, which can only be solved using a computer program.

**Systemic Models for Individual Elements**

After material enters the body through the lung or GI tracts, it may deposit energy in the compartments of these models, which are organs at risk for radiation exposure, but the material may as well be absorbed into the systemic circulation and deposited in various organs of the body and then eliminated. Thus the organs in which the activity is deposited, as well as the excretory organs will be exposed to radiation dose from particulate radiations, and all organs and tissues may be exposed to a significant degree from photons originating in the organs where the activity is concentrated (the lung, GI, and other important organs with radionuclide uptake). As discussed above, the characterization of the kinetics within any organ is often made with a quantity called the *biological half-time*, as the clearance of materials from organs of the body tends to be first-order, just like radioactive decay, and thus treatable with an exponential model. When first-order biological clearance (biological half-time) is combined with the radioactive decay of the radionuclide (physical half-time), the effective half-time (as defined above) is obtained.

**Biological Models for the Elements**

Biological models for the different elements of concern to occupational dose assessment are derived based on whatever available literature values may be found, from experiments involving animals or measurements in human subjects. Some of the simplest models involve an element being uniformly distributed in the whole-body and being eliminated with one or more exponential terms. For example, tritium ($^3$H) was characterized for years by a uniform whole-body distribution with a single exponential clearance component.
with a biological (and also effective) half-time of 10 days. More recently, a longer-lived component, called Organically Bound Tritium (OBT) has been defined with one or more longer half-times.

The model for cesium is treated as a uniform whole-body model with two effective half-times. Ten percent of the material is seen as clearing the body with a 2 day biological half-time and the other ninety percent with a 110 day biological half-time. It is not uncommon, in the whole-body or organs, to have material cleared with more than one half-time. Sometimes there is a reasonable and understandable reason for this (e.g., some material is cleared by systemic processes, having to do with blood flow and other material with clearance processes within the organ, after uptake and release by the cells). In other cases, it may not be entirely clear why there are multiple phases of clearance, but it is simply observed during the collection of biokinetic data.

Most of the biokinetic models in ICRP 30, which form the basis for the current U.S. regulatory structure for protecting workers from intakes of radionuclides, are of the form in which material moves from the “transfer compartment” (basically, the systemic circulation) to different organs, where it is eliminated with one or more biological half-times. For example, Figure 10.9 shows the ICRP 30 model for uranium. In this model, 54% of the activity that reaches the circulation is excreted quickly (the ICRP 30 model uses a biological half-time of 0.25 d), then about 22% goes to bone, where 20% is excreted with a 20 d biological half-time and the rest with a 5000 d biological half-time.

Kidneys and other tissues receive a little over 12% of the activity, with most of this being excreted with a 6 d biological half-time and a small fraction being retained with a 1500 d half-time. One might be tempted to ignore this small fraction in the dose calculations, but recall that the number of disintegrations is given by the product of the initial activity in a region and the effective half-time. So here, for example, $0.12 \times 6 = 0.72$ and $0.00052 \times 1500 = 0.78$, so these two components will contribute almost equally to the total number of disintegrations occurring in these regions. This kind of model, also sometimes called a “once through” model, is not terribly realistic, although it probably gives a reasonably good estimate of the number of disintegrations occurring.
10.4 Internal Dose Calculations for Radiation Workers

Figure 10.10 ICRP 69 systemic model for uranium. (Reprinted from Ref. 12 with permission from Elsevier.)

in each of the important source regions. This kind of model, at least for the more complicated cases, is being replaced by more realistic models in which material recirculation occurs, as in the example below, from ICRP 69\textsuperscript{12} for uranium (Figure 10.10).

Calculation of Organ Doses

Once we have completely characterized the biokinetic model for an element (including the kinetics of the intake component, lung or GI), we have the $U_S$ values for all of the important source regions, and just need to multiply them by the appropriate dose conversion factors ($SEE$s), as shown above. To calculate an $SEE$, one needs decay data, data on standard organ masses, and the “absorbed fractions” for photons. Decay data are not difficult to find. For electrons, we assume that the absorbed fraction is 1.0 for an organ irradiating itself and 0.0 for irradiation of other organs. Electrons have short ranges in human tissue relative to the dimensions of the organs, whereas photons will travel and scatter throughout all tissues of the body, and some photon energy will likely escape the body entirely. Absorbed fractions for photons are calculated using Monte Carlo radiation transport simulation methods in anthropomorphic phantoms, that is, mathematical representations of the human body.

The sizes of the organs are based on values given by the ICRP for standard reference persons, so we now have all of the pieces needed to calculate our $SEE$ values. The first complete descriptions of a phantom representing the reference adult were given in MIRD Pamphlets 5 and 5, Revised\textsuperscript{13,14} (Figure 10.11). These absorbed fractions were used to develop the $SEE$ values in ICRP 30\textsuperscript{5}. 

Figure 10.10 ICRP 69 systemic model for uranium. (Reprinted from Ref. 12 with permission from Elsevier.)
as well as the $S$ values in MIRD Pamphlet No. 11 \(^\text{15}\) (we study the MIRD system next). An improved set of absorbed fractions for a slightly different adult phantom and for five other individuals representing children of different ages (newborns, 1-yr-olds, 5-yr-olds, 10-yr-olds, 15-yr-olds) was published by Cristy and Eckerman in 1987. \(^\text{16}\) Then, in 1995, four phantoms representing the adult female, both nonpregnant and at three stages of pregnancy were published by Stabin et al. \(^\text{17}\) Before 1995, the Cristy and Eckerman 15-yr-old phantom was often used to represent the adult female. The Stabin et al. adult female phantom is somewhat different from the Cristy–Eckerman model. Others have as well proposed more detailed models of some organs, including the brain, \(^\text{18,19}\) eye, \(^\text{20}\) peritoneal cavity, \(^\text{21}\) prostate gland, \(^\text{22}\) and others.

**Example**

Ignoring organically bound tritium for the moment, we can calculate the number of transformations occurring from a 1 Bq intake of $^3$H, using the standard model that has a uniform, whole-body distribution with a biological retention half-time of 10 days. As the physical half-life of $^3$H is 12.3 years, the biological half-time is equal to the effective half-time. Thus, the number of
transformations is easily calculated as

\[ U_{\text{whole body}} = \frac{1}{\text{transformation}} \left( \frac{0.693}{10 \, \text{d} \times 86400 \, \text{s}} \right) = 1.2 \times 10^6 \text{ transformations} \]

The SEE for tritium in the whole body may be found to be \( 9 \times 10^{-8} \text{ MeV/g-transf} \). Thus, the committed dose from a 1 Bq intake is:

\[ H_{50} = 1.6 \times 10^{-10} \frac{\text{Sv g MeV}}{\text{MeV g-transf}} \times 9 \times 10^{-8} \frac{\text{MeV}}{\text{g-transf}} = 1.7 \times 10^{-11} \text{ Sv} \]

Thus our dose coefficient for \(^3\text{H}\) intakes is \( 1.7 \times 10^{-11} \text{ Sv/Bq intake} \). Given an intake of 1 kBq, we could immediately calculate an estimated committed dose of \( 1.7 \times 10^{-8} \text{ Sv} \).

For \(^{137}\text{Cs}\), as noted above, there are two components to the retention curve. 10% of the cesium in the body is cleared with a 2 day biological half-time and the rest with a 110 day biological half-time. To calculate the number of disintegrations resulting from the intake of 1 Bq of \(^{137}\text{Cs}\), we would need to calculate the effective half-times and apply the two fractions for the distribution:

\[
T_{\text{eff}-1} = \frac{2 \times 10960}{2 + 10960} \approx 2 \, \text{d}
\]
\[
T_{\text{eff}-2} = \frac{110 \times 10960}{110 + 10960} = 109 \, \text{d}
\]
\[
\lambda_{\text{eff}-1} = \frac{0.693}{2 \, \text{d}} = 0.346 \, \text{d}^{-1}
\]
\[
\lambda_{\text{eff}-2} = \frac{0.693}{109 \, \text{d}} = 0.0064 \, \text{d}^{-1}
\]

\[
U_{\text{whole body}} = \frac{0.1}{0.346 \, \text{d}^{-1}} + \frac{0.9}{0.0064 \, \text{d}^{-1}} = 0.29 + 141
\]

\[
= 141 \text{Bq} - d = 1.22 \times 10^7 \text{ transformations}
\]

\[
U_{\text{whole body}} = 1.443 \left[ 0.1 \times 2 \, \text{d} + 0.9 \times 109 \, \text{d} \right] = 0.29 + 141 = 141 \text{Bq} - d
\]

\[
= 1.22 \times 10^7 \text{ transformations}
\]

**Calculation of Permissible Intake Limits**

In the ICRP system, one is not content to simply calculate doses to individuals. In this system, we wish to take the calculation a step further and calculate the amount of activity that would give a certain dose (the permissible annual dose) from the dose conversion coefficients for a given nuclide. The dose conversion factors are usually calculated as 50-year committed dose (Sv) per Bq of intake (by inhalation or ingestion). Knowing our annual dose limits in Sv, we can thus calculate the number of Bq that are permissible to take in during one year of work. The only complication is that we really have two dose limits that must be satisfied at the same time: the stochastic limit (50 mSv effective whole-body) and the nonstochastic limit (500 mSv to any organ). We resolve this by calculating the permissible amount of activity that will satisfy both limits and choose the smaller of the two values as our controlling limit.
Example

Calculate the ALI and the DAC for inhalation of class D $^{32}$P. Solution of the biokinetic model gives the following values for the number of disintegrations in various source organs for a 1 Bq intake.

- Lungs: $1.8 \times 10^4$
- ULI contents: $1.4 \times 10^3$
- LLI contents: $2.5 \times 10^3$
- Cortical bone: $1.5 \times 10^5$
- Trabecular bone: $1.5 \times 10^5$
- Other tissues: $2.8 \times 10^5$

We may find the dose conversion factors (SEEs) in ICRP 30, and solve for the $H_{50}$ values as follows.

\[
H_{50,\text{gonads}} = 1.6 \times 10^{-10} \frac{\text{Sv}}{\text{MeV}} \times \frac{2.8 \times 10^5 \text{ transf}}{\text{Bq intake}} \times \frac{9.9 \times 10^{-6} \text{ MeV}}{\text{g transf}}
\]

\[
= 4.4 \times 10^{-10} \text{ Sv Bq}^{-1}
\]

\[
H_{50,\text{breasts}} = 1.6 \times 10^{-10} \frac{\text{Sv}}{\text{MeV}} \times \frac{2.8 \times 10^5 \text{ transf}}{\text{Bq intake}} \times \frac{9.9 \times 10^{-6} \text{ MeV}}{\text{g transf}}
\]

\[
= 4.4 \times 10^{-10} \text{ Sv Bq}^{-1}
\]

\[
H_{50,\text{marrow}} = 1.6 \times 10^{-10} \frac{\text{Sv}}{\text{MeV}} \times \frac{2.8 \times 10^5 \text{ transf}}{\text{Bq intake}} \times \frac{9.9 \times 10^{-6} \text{ MeV}}{\text{g transf}}
\]

\[
+ 1.6 \times 10^{-10} \frac{\text{Sv}}{\text{MeV}} \times \frac{1.5 \times 10^5 \text{ transf}}{\text{Bq intake}} \times \frac{2.3 \times 10^{-4} \text{ MeV}}{\text{g transf}}
\]

\[
= 6.0 \times 10^{-9} \text{ Sv Bq}^{-1}
\]

\[
H_{50,\text{lungs}} = 1.6 \times 10^{-10} \frac{\text{Sv}}{\text{MeV}} \times \frac{2.8 \times 10^5 \text{ transf}}{\text{Bq intake}} \times \frac{9.9 \times 10^{-6} \text{ MeV}}{\text{g transf}}
\]

\[
+ 1.6 \times 10^{-10} \frac{\text{Sv}}{\text{MeV}} \times \frac{1.8 \times 10^4 \text{ transf}}{\text{Bq intake}} \times \frac{6.9 \times 10^{-4} \text{ MeV}}{\text{g transf}}
\]

\[
= 2.4 \times 10^{-9} \text{ Sv Bq}^{-1}
\]

\[
H_{50,\text{ULI}} = 1.6 \times 10^{-10} \frac{\text{Sv}}{\text{MeV}} \times \frac{2.8 \times 10^5 \text{ transf}}{\text{Bq intake}} \times \frac{9.9 \times 10^{-6} \text{ MeV}}{\text{g transf}}
\]

\[
+ 1.6 \times 10^{-10} \frac{\text{Sv}}{\text{MeV}} \times \frac{1.4 \times 10^3 \text{ transf}}{\text{Bq intake}} \times \frac{1.6 \times 10^{-3} \text{ MeV}}{\text{g transf}}
\]

\[
= 8.0 \times 10^{-10} \text{ Sv Bq}^{-1}
\]

\[
H_{50,\text{LLI}} = 1.6 \times 10^{-10} \frac{\text{Sv}}{\text{MeV}} \times \frac{2.8 \times 10^5 \text{ transf}}{\text{Bq intake}} \times \frac{9.9 \times 10^{-6} \text{ MeV}}{\text{g transf}}
\]

\[
+ 1.6 \times 10^{-10} \frac{\text{Sv}}{\text{MeV}} \times \frac{2.5 \times 10^3 \text{ transf}}{\text{Bq intake}} \times \frac{2.6 \times 10^{-3} \text{ MeV}}{\text{g transf}}
\]

\[
= 1.5 \times 10^{-9} \text{ Sv Bq}^{-1}
\]
Having the individual $H_{50}$ values, we can choose the appropriate tissue weighting factors for each organ and also calculate the effective dose equivalent:

<table>
<thead>
<tr>
<th>Organ</th>
<th>$H_{50}$</th>
<th>$w_T$</th>
<th>$w_T \times H_{50,T}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gonads</td>
<td>$4.4 \times 10^{-10}$</td>
<td>0.25</td>
<td>$1.1 \times 10^{-10}$ Sv/Bq</td>
</tr>
<tr>
<td>Breast</td>
<td>$4.4 \times 10^{-10}$</td>
<td>0.15</td>
<td>$6.6 \times 10^{-11}$ Sv/Bq</td>
</tr>
<tr>
<td>Red marrow</td>
<td>$6.0 \times 10^{-9}$</td>
<td>0.12</td>
<td>$7.2 \times 10^{-10}$ Sv/Bq</td>
</tr>
<tr>
<td>Lungs</td>
<td>$2.4 \times 10^{-9}$</td>
<td>0.12</td>
<td>$2.9 \times 10^{-10}$ Sv/Bq</td>
</tr>
<tr>
<td>Bone surface cells</td>
<td>$5.9 \times 10^{-9}$</td>
<td>0.03</td>
<td>$1.8 \times 10^{-10}$ Sv/Bq</td>
</tr>
<tr>
<td>ULI</td>
<td>$8.0 \times 10^{-10}$</td>
<td>0.06</td>
<td>$4.8 \times 10^{-11}$ Sv/Bq</td>
</tr>
<tr>
<td>LLI</td>
<td>$1.5 \times 10^{-9}$</td>
<td>0.06</td>
<td>$9.0 \times 10^{-11}$ Sv/Bq</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\Sigma = 1.5 \times 10^{-9}$ Sv/Bq</td>
</tr>
</tbody>
</table>

Now we need to calculate our two possible intake values (called Annual Limits on Intake, or ALIs). For the stochastic $ALI$, we divide the stochastic dose limit into the effective dose (the sum of the right hand column). For the nonstochastic $ALI$, we divide the nonstochastic limit into the highest of the dose/intake values for the individual organs:

$$ALI_{\text{stochastic}} = \frac{0.05 \frac{Sv}{y}}{1.5 \times 10^{-9} \frac{Sv}{Bq}} = 3.3 \times 10^7 \frac{Bq}{y}$$

$$ALI_{\text{non-stochastic}} = \frac{0.5 \frac{Sv}{y}}{6.0 \times 10^{-9} \frac{Sv}{Bq}} = 8.3 \times 10^7 \frac{Bq}{y}$$

As the stochastic limit is less than the nonstochastic limit, this becomes the limiting value, and is our actual $ALI$ ($3.3 \times 10^7$ Bq).

**Calculation of Permissible Air Concentrations**

Now, for the calculation of an air concentration that may be present all year that a worker may breathe without exceeding the dose limit, we just divide the chosen $ALI$ by the amount of air breathed in a year ($2400$ m$^3$, based on a breathing rate of 0.02 m$^3$/min):

$$DAC = \frac{3.3 \times 10^7 \text{ Bq}}{2400 \text{ m}^3} = 1.4 \times 10^4 \text{ Bq m}^{-3}$$
Important notes:

- If one takes in exactly one $ALI$ of any nuclide, one is exposed exactly at the dose limit, and may have no other sources of exposure during that year. Thus, the true compliance equation is:

$$\sum \frac{\text{Intake}_i}{ALI_i} + \frac{H_{ext}}{50 \text{ mSv}} \leq 1.0$$

- The DAC gives the concentration that may be present continuously throughout a (2000 hr) working year. Thus, the true limit on air concentrations is based on the idea of DAC-hours: if one is exposed to 2000 DAC-hours in a year, one takes in exactly 1 $ALI$ by inhalation, and thus is exposed exactly at the dose limit. Another form of the compliance equation thus may be:

$$\sum \frac{\text{Intake}_i}{ALI_i} + \sum \frac{\text{DAC-hours}}{2000} + \frac{H_{ext}}{50 \text{ mSv}} \leq 1.0$$

One thus may be exposed to a level of 2000 DACs for 1 hour and still be within permissible dose limits. This again assumes that the individual had no external radiation exposures during the year, and had no other intakes, either by inhalation or ingestion, during that year.

### 10.5 Internal Dose Calculations for Nuclear Medicine Patients

Just as for radiation workers, to calculate radiation dose for nuclear medicine patients, one needs a kinetic model, based on measurements made in animal or human studies, and dose conversion factors. The kinetic data are obtained from animal or human studies, as for radiation workers. However, radionuclides in nuclear medicine are bound to a very wide variety of compounds, and a separate kinetic model must be developed for each compound. To obtain approval from the Food and Drug Administration to distribute a new radiopharmaceutical, a company must show that the drug is both safe and efficacious. Safety concerns include, but are not limited to, radiation doses expected to be received by patients who receive the radiopharmaceutical. Dose conversion factors are available for all of the standard models, as they are for radiation workers; in fact, the same anthropomorphic models are used. Kinetic models must be derived for each new compound, which involves the proper design and execution of experiments in either animals or humans to obtain the necessary data to build a kinetic model.

**Kinetic Data**

To design and execute a good kinetic study, one needs to collect the correct data, enough data, and express the data in the proper units. The basic data needed are the fraction (or percentage) of administered activity in important source organs and excreta samples. We discuss later how these data are gathered from an animal or human study. It is very important, in either type of study, to take enough samples to characterize both the distribution
and retention of the radiopharmaceutical over the course of the study. The following criteria are essential.

- Catch the early peak uptake and rapid washout phase.
- Cover at least three effective half-times of the radiopharmaceutical.
- Collect at least two time points per clearance phase.
- Account for 100% of the activity at all times.
- Account for all major paths of excretion (urine, feces, exhalation, etc.).

Some knowledge of the expected kinetics of the pharmaceutical are needed for a good study design. For example, the spacing of the measurements and the time of the initial measurement will be greatly different if we are studying a $^{99m}$Tc labeled renal agent that is 95% cleared from the body in 180 minutes or an $^{131}$I labeled antibody that clears about 80% in the first day and the remaining 20% over the next two weeks. A key point that researchers can overlook is the characterization of excretion. Very often, the excretory organs (intestines, urinary bladder) are the organs that receive the highest absorbed doses, as 100% of the activity (minus decay) will eventually pass through one or both of these pathways at different rates. If excretion is not quantified, the modeler must make the assumption that the compound is retained in the body and removed only by radioactive decay. For very short-lived radionuclides, this may not be a problem and in fact may be quite accurate. For moderately long-lived nuclides, this can cause an overestimate of the dose to most body organs and an underestimate of the dose to the excretory organs, perhaps significantly.

**Development of Kinetic Data**

Data for kinetic studies are generally derived from one of two sources:

- Animal studies, usually performed for submission of an application for approval to the Food and Drug Administration for use of a so-called Investigational New Drug (IND)
- Human studies, usually performed in Phase I, II, or III of approval of a New Drug Application (NDA)

**Animal Studies**

In an animal study, the radiopharmaceutical is administered to a number of animals which are then sacrificed at different times, and the organs harvested and counted for activity (or perhaps imaged). The extrapolation of animal data to humans is not an exact science. One method of extrapolating animal data is the % kg/g method. In this method, the animal organ data need to be reported as percent of injected activity per gram of tissue, and the animal whole-body weight must be known. The extrapolation to humans then uses the human organ and whole-body weight, as follows.

\[
\left( \frac{\%}{\text{organ}} \right)_{\text{animal}} \times (\text{kg}_{\text{TB}} \text{ weight})_{\text{animal}} \times \left( \frac{\text{g}_{\text{organ}}}{\text{kg}_{\text{TB}} \text{ weight}} \right)_{\text{human}} = \left( \frac{\%}{\text{organ}} \right)_{\text{human}}
\]

**Human Studies**

In human studies, data are collected with a nuclear medicine gamma camera. Quantification of data gathered with these cameras is rather involved. The gamma camera, also called an “Anger” camera, relies on the basic principles and design of inventor Hal Anger. Briefly, the camera employs a single large
sheet of scintillation material, coupled (usually via a light guide) to a group of
photomultiplier tubes, with a collimator between the patient and scintillator.
Collimators are typically made of lead, are about 4 to 5 cm thick and 20
by 40 cm on a side. The collimator contains thousands of square, round,
or hexagonal parallel holes through which gamma rays are allowed to pass,
supposedly photons coming from angles other than parallel to the camera are
attenuated (but this is not strictly true: septal penetration is an important source
of noise in gamma camera images). There are many designs of collimators, to
handle low-, medium-, or high-energy nuclides on different systems.

Some solid-state gamma cameras are under development, but the majority
in use today use scintillation technology. Because the photomultiplier tubes
are large compared to the desired final image resolution, a signal-weighted
average of at least several of the photomultipliers is used to estimate the actual
interaction location in the scintillator. A projection image of the object under
study (organs within a patient) is thus developed after many thousands or
millions of events strike the crystal and are processed. As this is a projection
image, the actual depth of the object within the patient is not known. Also,
the image contains events caused by Compton scattered photons, photons
from energies other than that of the photopeak of interest, and possibly other
interferences.

The number of counts at individual points across the Field Of View (FOV) is
provided directly by the gamma camera computer. The FOV is usually a square
field, with 128 × 128, 256 × 256, or some other number of points (pixels) in
the view. One may draw Regions Of Interest (ROIs) around images of objects
that will be recognizable as internal organs or structures; the number of counts
in a ROI, however, cannot be used directly to calculate how much activity is
in the organ. A number of corrections are needed to the observed number of
counts to obtain a reliable estimate of activity in this object.

Depth Dependence
To remove uncertainties about the depth of the object, usually images are taken
in front of and behind the patient, and a geometric mean of the two values is
taken. This geometric mean has been shown to be relatively independent of
depth for most radionuclides of interest, and thus this quantity is thought to be
the most reliable for use in quantification.

Attenuation
Photons leaving the object and striking either detector will have been attenu-
at ed within the patient to some degree. Each setup will have slightly different
characteristics, depending on the camera itself, the collimator employed, and
the nuclide of interest. A study to establish the attenuation coefficient for
each combination must be done before collecting data. Patient thickness in a
region where activity is to be quantified is estimated by using a sheet, or flood,
source of a particular nuclide (often $^{57}$Co), with knowledge of this nuclide’s
attenuation characteristics in the same setup.

Scatter
Gamma cameras can collect counts in each pixel within a user-defined energy
“window,” which is generally set to be ±15% or 20% of the photopeak energy
(e.g., 140 keV ± 28 keV). If the photopeak is the highest energy in the
radionuclide’s emission spectrum (as is true for $^{99m}$Tc, but not for $^{131}$I), scatter
events in the photopeak window will be due to small angle scattering, which can be approximately corrected by subtracting counts from an equal width window below the photopeak area. If there are other high-energy events that may contribute to photopeak counts, another window is needed above the photopeak window to correct for these events. Other, mathematical, techniques (convolution-based) have been suggested (e.g., Floyd et al.\textsuperscript{26}) in addition to double- or triple-energy window techniques; other authors developed a method to correct for scatter by evaluating depth-dependent buildup factors for objects of different size. After scatter correction has been applied, the activity of the source within the ROI is thus given by

\[ A_{\text{ROI}} = \sqrt{\frac{I_A I_P f_j}{e^{-\mu_e t} C}} \]

where \( I_A \) and \( I_P \) are the anterior and posterior counts in the region, \( \mu_e \) is the effective attenuation coefficient, \( t \) is the average patient thickness over the ROI, \( f_j \) is the source self-attenuation coefficient (given as \( [((\mu_e t/2)/\sinh(\mu_e t/2))] \), but which is rarely of much impact in the calculation and so is usually neglected), and \( C \) is a source calibration factor (cts/s per Bq), obtained by counting a source of known activity in air.

Thus, activity in identifiable regions of the body, such as the liver, spleen, kidneys, and so on may be determined at individual times. ROIs may also be drawn over the entire body, to track the retention and excretion of the compound in the body. Excreta samples may also be taken to study excretion pathways. If only a single excretion pathway is important, knowledge of whole-body clearance may be used to explain excretion.

**Tomography**

In the derivation given above, the gamma camera is assumed to stay in a fixed position with respect to the patient, with image data gathered in the anterior and posterior camera heads and then processed. The projection image thus obtained gives a two-dimensional (2D), or planar, image of the patient. This may be perfectly adequate for gathering the diagnostic information needed or for calculating dose estimates. If the camera heads are rotated around the body, with projection images taken at multiple angles around the body, 3D image information may be reconstructed from these multiple images, using techniques common to tomographic imaging in other modalities (such as CT). This 3D information may thus be displayed in slices or 3D renderings, yielding a more detailed understanding of the activity distribution.

Single Photon Emission Computed Tomography (SPECT) is applied to most radionuclides, whereas Positron Emission Tomography (PET) applies to positron-emitting nuclides. Projection images are taken at many angles around the body, typically every 3 or 6° in a 360° range. Each projection is then “back-projected” across the entire field; as areas intersect from different projections in an individual “slice” of the body in space they reinforce, giving a 2D image in that slice of the object (Figure 10.12). Direct back-projection introduces artifacts into the reconstructed images, so back-projection is usually performed by applying a filter to the data as they are back-projected.

It is convenient in dealing with the volumes of data involved in these calculations to convert information from the spatial domain into the frequency domain by taking the Fourier transform of the data. Small objects in the spatial
Figure 10.12 Reconstruction of image data from tomography.

domain correspond to high frequencies in the frequency domain. Data filters are used to try to eliminate as much noise and background from the images as possible without removing true information from the object. Many types of filters have been developed, and most have adjustable parameters that can be manipulated to obtain the best estimates possible of the 3D image for use in diagnosis or activity quantification.

With a filtered reconstructed image, one has still not corrected for attenuation. Every “voxel” (3D correlate of a 2D “pixel”) in a given slice will have been back-projected through a different amount of tissue (of variable attenuation characteristics, perhaps including soft tissue, bone, or lung) from the original projection image pixel. One may assume an approximately uniform attenuation field for the body, or, if possible, an attenuation “map” may be defined by using a CT image of the same patient prior to SPECT imaging. By using multiple projections and studying the detected radiation at detectors on the opposite side of the patient, attenuation characteristics of each voxel in the to-be reconstructed image field can be defined. Then, by successive iterative approximations, one may attempt to obtain an optimized 3D image that has been corrected for attenuation (scatter may also have been removed by methods similar to that used in the planar approach above). This is the approach taken with the Maximum Likelihood Expectation Maximization (MLEM) or Ordered Sets Expectation Maximization (OSEM) reconstruction algorithm, which has a coupled CT/SPECT camera system so that all the needed data can be obtained without moving the patient and having to deal with issues of image registration.

Figure 10.12 illustrates the idea of use of an iterative reconstruction algorithm for attenuation correction. Every voxel \( j \) in the reconstruction matrix \((m \times m)\) has an attenuation factor for a given projection angle and detector element (1 through \( n \)), depending on the number of elements between them, the cross-section of each element in the projection, and the material composition of each element.

Even with the best possible images, however, the edges of the object image will not perfectly correspond to those of the true object, because of the inherent
resolution limits of the gamma camera system (typically of the order of 10s of mm currently for most nuclides and collimators). Studies must be undertaken with objects of known mass and activity, in scattering media with and without background activity, to evaluate volume and activity recovery coefficients for a range of object sizes and activity contrast values \((\text{source} - \text{bkgd})/\text{source}\) anticipated in practice. Patient motion can also interfere with the quality of image data.

**Analysis of Kinetic Data**

Let us now assume that we have gathered a series of measurements that represent either retention and/or excretion. Our task is to interpret these measurements in such a way as to derive a workable kinetic model that can be used to estimate \(\dot{A}\) for each significant source region in our system. In general, there are three levels of complexity that our analysis can take.

**Direct Integration**

One can directly integrate under the actual measured values by a number of methods. This does not give very much information about your system, but it does allow you to calculate \(\tau\) rather easily. The most common method used is the trapezoidal method, simply approximating the area by a series of trapezoids.

**Least Squares Analysis**

An alternative to brute-force integration of the data is to attempt to fit curves of a given shape to the data. The curves are represented by mathematical expressions that can be directly integrated. The most common approach is to attempt to characterize a set of data by a series of exponential terms, as many systems are well represented by this form, and exponential terms are easy to integrate. In general, the approach is to minimize the sum of the squared distance of the data points from the fitted curve. The curve will have the form:

\[
A(t) = a_1 e^{-b_1 t} + a_2 e^{-b_2 t} + \cdots
\]

The method looks at the squared difference between each point and the solution of the fitted curve at that point, and minimizes this quantity by taking the partial derivative of this expression with respect to each of the unknowns \(a_i\) and \(b_i\) and setting it equal to zero. Once the ideal estimates of \(a_i\) and \(b_i\) are obtained, the integral of \(A(t)\) from zero to infinity is simply:

\[
\int_{0}^{\infty} A(t)\,dt = \frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots
\]

If the coefficients \(a_i\) are in units of activity, this integral represents cumulated activity (the units of the \(b_i\) are time\(^{-1}\)).

**Compartmental Models**

The situation frequently arises wherein you either know quite a bit about the biological system under investigation or you would like to know in greater detail how this system is working. In this case, you can describe the system as a group of compartments linked through transfer rate coefficients. Solving for \(\dot{A}\) of the various compartments involves solving a system of coupled differential equations describing transfer of the tracer between compartments and
elimination from the system. The solution to the time activity curve for each compartment will usually be a sum of exponentials, but not obtained by least squares fitting each compartment separately, but by varying the transfer rate coefficients between compartments until the data are well fit by the model.

**Dose Calculations**

Dose calculations in the MIRD system are completely identical to those in the ICRP 30 system, we just use different symbols:

\[ U_s \Rightarrow \tilde{A} \]
\[ SEE \Rightarrow S \]
\[ D_1 = \tilde{A}_1 S(1 \leftarrow 1) + \tilde{A}_2 S(1 \leftarrow 2) \]
\[ D_2 = \tilde{A}_1 S(2 \leftarrow 1) + \tilde{A}_2 S(2 \leftarrow 2) \]
\[ D_3 = \tilde{A}_1 S(3 \leftarrow 1) + \tilde{A}_2 S(3 \leftarrow 2) \]

Looking up these values in tables can make the calculation of internal doses a not-too-painful exercise. Software programs have shortcut the process even further.

**Example: First Principles**

ICRP Report #23 describes Reference Man as containing 140 grams of potassium. Estimate the average beta dose rate in rad/week to the whole body, given the following information.

<table>
<thead>
<tr>
<th>Reference Man</th>
<th>70,000 g</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{40}\text{K}) in potassium</td>
<td>0.012%</td>
</tr>
<tr>
<td>(^{40}\text{K}) beta decay probability</td>
<td>90%</td>
</tr>
<tr>
<td>(^{40}\text{K}) maximum beta energy</td>
<td>1.3 MeV</td>
</tr>
<tr>
<td>(^{40}\text{K}) half-life</td>
<td>(1.2 \times 10^9) years</td>
</tr>
<tr>
<td>Avogadro’s number</td>
<td>(6.025 \times 10^{23})</td>
</tr>
<tr>
<td>Energy conversion</td>
<td>(1.6021 \times 10^{-6}) erg/MeV</td>
</tr>
<tr>
<td>Dose conversion</td>
<td>100 erg/g = 1 rad</td>
</tr>
</tbody>
</table>

We use no particular system to evaluate this dose rate, just the principle that absorbed dose is energy absorbed per unit mass. Here we have a uniform distribution of radioactivity in 70,000 g of material, and we just want to estimate the beta dose rate. First, we need to find how much activity we have, which is calculated from the number of grams, from the well-known relationship:

\[ A = \lambda N \]

In this problem:

\[
A = \left( \frac{0.693}{1.2 \times 10^9 \text{ yr}} \right) \left( \frac{1 \text{ yr}}{\pi \times 10^7 \text{ sec}} \right) (140 \text{ g K}) \left( \frac{1.2 \times 10^{-4} \text{ g K} - 40}{\text{g K}} \right) \\
\times \left( \frac{\text{mol K} - 40}{40 \text{ g K} - 40} \right) \left( \frac{6.025 \times 10^{23} \text{ atoms}}{\text{mol}} \right)
\]

\[ A = 4.65 \times 10^5 \text{ atoms/sec} = 4.65 \times 10^5 \text{ dis/sec} \]
We can directly calculate the dose rate in the object of 70,000 g now, because we know the activity. Dose rate is the product of the activity, the average energy of decay, the abundance of decay, the absorbed fraction, and the factor \( k \) (which just converts units), divided by the mass. There is a small trick in this problem, in that I gave you the maximum beta energy for \(^{40}\text{K}\); you must use the average energy to get the average dose rate. If you can look this up in a decay data book it would be better; here we just use the rule of thumb that the average beta energy is one-third the maximum.

\[
D = \left( \frac{4.65 \times 10^3 \text{ dis}}{\text{sec}} \right) \left( \frac{6.048 \times 10^5 \text{ sec}}{\text{week}} \right) \left( \frac{1.3 \text{ MeV}}{3 \beta} \right) \left( \frac{0.9 \beta}{\text{dis}} \right) \\
\times \left( \frac{1.602 \times 10^{-6} \text{ erg}}{\text{MeV}} \right) \left( \frac{\text{g} - \text{rad}}{100 \text{ erg}} \right) \left( \frac{1}{70000 \text{ g}} \right)
\]

\[
D = 2.5 \times 10^{-4} \text{rad/week}
\]

In this problem, \( k \) is the product of the number of seconds in a week, the \( \text{erg/MeV} \) over the factor of 100 \( \text{erg/g-rad} \). The first term in the dose rate equation is the activity we calculated above, the third term is the average energy per decay (1.3/3), and the next term (0.9) is the abundance. The absorbed fraction is 1.0, and is given in the same term as the mass (70,000 g). This is a pretty accurate analysis of the beta dose rate that you get every week from the natural \(^{40}\text{K}\) in your body. Of course there might be a bit less or a bit more activity in your body, you might weigh a bit less or more than 70 kg, and so on. And if you wanted to study the total dose rate, you would need to evaluate the gamma component, too, the methods for which we look at in other examples.

**Example: Contributions to Organ Dose**

Calculate the dose to liver, spleen, and lungs from \(^{90}\text{Y}\) activity in the liver and spleen as described below. To understand the contributions to the total dose to any organ, consider separately:

1. Dose from nonpenetrating radiations (nonpenetrating self-dose)
2. Dose from penetrating radiations when source and target are the same organ (penetrating self-dose)
3. Dose from penetrating radiations when source and target are not the same organ (cross-irradiation dose)

<table>
<thead>
<tr>
<th>Organ</th>
<th>Mass (kg)</th>
<th>( \bar{A} ) (( \mu \text{Ci-hr} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liver</td>
<td>1.91</td>
<td>2000</td>
</tr>
<tr>
<td>Spleen</td>
<td>0.183</td>
<td>2000</td>
</tr>
<tr>
<td>Lungs</td>
<td>1.00</td>
<td>0</td>
</tr>
</tbody>
</table>

For \(^{90}\text{Y}\):

\[
\Delta_p = 0.000 \text{ g-rad/} \mu \text{Ci-hr}
\]

\[
\Delta_{np} = 1.99 \text{ g-rad/} \mu \text{Ci-hr}
\]

\((p = \text{penetrating}, \ np = \text{nonpenetrating})\)
Yttrium-90 has no penetrating component. Absorbed fractions for any organ for nonpenetrating emissions are one and for any other organ are zero. The organ doses, in rad, are

<table>
<thead>
<tr>
<th>Source Organ</th>
<th>Liver</th>
<th>Spleen</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liver</td>
<td>NA</td>
<td>NA</td>
<td>2.1</td>
</tr>
<tr>
<td>Spleen</td>
<td>0</td>
<td>NA</td>
<td>22</td>
</tr>
<tr>
<td>Lungs</td>
<td>0</td>
<td>0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The liver dose is \((2000 \mu \text{Ci} \cdot \text{hr} \times 1.99 \text{ g-rad}/\mu \text{Ci} \cdot \text{hr})/1910 \text{ g} = 2.1 \text{ rad}\).
The spleen dose is \((2000 \mu \text{Ci} \cdot \text{hr} \times 1.99 \text{ g-rad}/\mu \text{Ci} \cdot \text{hr})/183 \text{ g} = 22 \text{ rad}\).
The lung dose is of course zero, as the lung had no assigned activity, and activity in liver and spleen were assumed not to irradiate lung.

**Teaching Points**

1. Liver and spleen had equal cumulated activities. Because spleen is about \(10 \times\) smaller, the dose is \(10 \times\) higher (exactly true for beta emitters, results will vary for gamma emitters, but the trend will be the same).
2. Units, units, units. The mass was given in kg; we needed to convert it to g. If we didn’t, our answers were off by a factor of 1000, and our calculations would have wrongly indicated that the patient may have received an extremely high dose.

**Example: Contributions to Organ Dose**

Now, repeat the same problem for \(^{99m}\text{Tc}\).

Complete the data tables below to calculate the doses. Consider 140 keV to be the only penetrating decay energy.

<table>
<thead>
<tr>
<th>Organ</th>
<th>Mass (kg)</th>
<th>(\dot{A}) ((\mu\text{Ci-hr}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liver</td>
<td>1.91</td>
<td>2000</td>
</tr>
<tr>
<td>Spleen</td>
<td>0.183</td>
<td>2000</td>
</tr>
<tr>
<td>Lungs</td>
<td>1.00</td>
<td>0</td>
</tr>
</tbody>
</table>

For \(^{99m}\text{Tc}\):

\(\Delta_p = 0.267 \text{ g-rad}/\mu\text{Ci-hr}\)
\(\Delta_{np} = 0.0342 \text{ g-rad}/\mu\text{Ci-hr}\)

\((p = \text{penetrating}, np = \text{nonpenetrating})\)
Absorbed Fractions*

<table>
<thead>
<tr>
<th>Target Organ (rT)</th>
<th>( \phi (rT \leftarrow \text{liver}) )</th>
<th>( \phi (rT \leftarrow \text{spleen}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liver</td>
<td>0.162</td>
<td>0.0071</td>
</tr>
<tr>
<td>Spleen</td>
<td>0.0062</td>
<td>0.072</td>
</tr>
<tr>
<td>Lungs</td>
<td>0.0098</td>
<td>0.0085</td>
</tr>
</tbody>
</table>

* Absorbed fractions from reference 9

<table>
<thead>
<tr>
<th>Source Organ</th>
<th>Liver</th>
<th>Spleen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target Organ</td>
<td>( p )</td>
<td>( np )</td>
</tr>
<tr>
<td>Liver</td>
<td>0.045</td>
<td>0.036</td>
</tr>
<tr>
<td>Spleen</td>
<td>0.0018</td>
<td>0</td>
</tr>
<tr>
<td>Lungs</td>
<td>0.0052</td>
<td>0</td>
</tr>
</tbody>
</table>

Teaching Points
1. Again, with equal cumulated activities in liver and spleen, roughly a tenfold difference in dose, due to the mass difference.
2. Now lung gets a dose, even with no assigned activity. This is due to penetrating dose from the liver and spleen, that is, photons that originate in these organs but deposit dose in the lungs.

Example: Calculation of \( S \)-Value for Average Organ Dose
We can calculate an \( S \)-value for liver self-irradiation from \( ^{99m}\text{Tc} \) by combining the appropriate decay data with calculated absorbed fractions. Table 10.3 shows the decay scheme for \( ^{99m}\text{Tc} \), from the RADAR Web site.

At first glance there appear to be a considerable number of emissions to consider. However, for our purposes, we can consider \( ^{99m}\text{Tc} \) to have only five emissions: one \( \gamma \) ray, three X-rays, and a group of nonpenetrating emissions. We can group the nonpenetrating emissions together because they are all multiplied by the same absorbed fraction (1.0), and so in the sum \( \Sigma n_i E_i \phi_i \), we may sum the \( n_i E_i \) and multiply the whole sum by \( \phi = 1.0 \). To calculate the \( S \)-value for liver irradiating itself, then, we need only to look up the appropriate absorbed fractions for the penetrating emissions, and sum over all emissions:

\[
\begin{array}{ccccccc}
\text{Emission} & \begin{array}{c} \begin{array}{c} n \\ E \end{array} \end{array} & k \Sigma n_i E_i & \phi & k \Sigma n_i E_i \phi_i \\
\gamma 2: & 0.891 & 0.1405 & 0.267 & 0.162 & 0.0432 \\
K_{\alpha 1} \text{x-ray} & 0.04 & 0.0184 & 0.0016 & 0.82 & 0.0013 \\
K_{\alpha 2} \text{x-ray} & 0.021 & 0.0182 & 0.0008 & 0.82 & 0.00066 \\
K_{\alpha 1} \text{x-ray} & 0.0068 & 0.0206 & 0.0003 & 0.78 & 0.00023 \\
\text{Nonpenetrating} & -- & -- & 0.0343 & 1.0 & 0.0343 \\
\end{array}
\]

TOTAL = 0.080
**Table 10.3** 99m-Tc-43 decay mode: IT half-life: 6.01 H.

<table>
<thead>
<tr>
<th>Emission Type</th>
<th>Mean Energy (MeV)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>ce-M e−</td>
<td>0.0016</td>
<td>0.7460</td>
</tr>
<tr>
<td>Auger-L e−</td>
<td>0.0022</td>
<td>0.1020</td>
</tr>
<tr>
<td>Auger-K e−</td>
<td>0.0155</td>
<td>0.0207</td>
</tr>
<tr>
<td>ce-K e−</td>
<td>0.1195</td>
<td>0.0880</td>
</tr>
<tr>
<td>ce-L e−</td>
<td>0.1216</td>
<td>0.0055</td>
</tr>
<tr>
<td>Auger-K e−</td>
<td>0.1375</td>
<td>0.0107</td>
</tr>
<tr>
<td>ce-N+ e−</td>
<td>0.1396</td>
<td>0.0017</td>
</tr>
<tr>
<td>ce-M e−</td>
<td>0.1400</td>
<td>0.0019</td>
</tr>
<tr>
<td>ce-M e−</td>
<td>0.1404</td>
<td>0.0004</td>
</tr>
<tr>
<td>ce-M e−</td>
<td>0.1421</td>
<td>0.0003</td>
</tr>
<tr>
<td>L X-ray</td>
<td>0.0024</td>
<td>0.0048</td>
</tr>
<tr>
<td>Kα1 X-ray</td>
<td>0.0183</td>
<td>0.0210</td>
</tr>
<tr>
<td>Kα2 X-ray</td>
<td>0.0184</td>
<td>0.0402</td>
</tr>
<tr>
<td>Kβ X-ray</td>
<td>0.0206</td>
<td>0.0120</td>
</tr>
<tr>
<td>γ</td>
<td>0.1405</td>
<td>0.8906</td>
</tr>
<tr>
<td>γ</td>
<td>0.1426</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

We set $k$ equal to 2.13, which causes the units on the third and fifth columns to be g-rad/$\mu$Ci-hr, given the energy in MeV. The $S$-value is simply the sum of the values in the fifth column divided by the mass of the liver, 1800 g:

$$S(\text{liver} \leftarrow \text{liver}) = \frac{0.080}{1800 \text{ g}} = 4.4 \times 10^{-5} \text{ rad}/\mu\text{Ci-hr}$$

**Example: Dose to One Organ**

Data extrapolated from an animal study yield the following parameters for a new compound tagged to $^{99m}\text{Tc}$.

Liver 
- $f_1 = 0.3$  
- $T_{e1} = 0.5$ hours  
- $f_2 = 0.1$  
- $T_{e2} = 5.5$ hours

Kidneys 
- $f = 0.2$  
- $T_e = 1.2$ hours

where $f$ is the fraction of injected activity (note that only 60% of the injected activity is accounted for by considering only these two organs). Let’s calculate the dose to the liver. If this were a real problem, we would calculate dose to the liver, kidneys, gonads, red marrow, and perhaps a few other organs. We find the following $S$-values in MIRD 11:

$$S(\text{liver} \leftarrow \text{liver}) = 4.6 \times 10^5 \text{ rad}/\mu\text{Ci-hr}$$

$$S(\text{liver} \leftarrow \text{kidneys}) = 3.9 \times 10^6 \text{ rad}/\mu\text{Ci-hr}$$

(The liver to liver $S$-value is slightly different than we had calculated, as MIRD 11 used slightly different decay data.) Assume $A_0 = 1$ mCi = 1000 $\mu$Ci; then,

$$\bar{A}(\text{liver}) = 1.443 \cdot 1000 \mu\text{Ci} \cdot (0.3 \cdot 0.5 \text{ hr} + 0.1 \cdot 5.5 \text{ hr}) = 1010 \mu\text{ Ci-hr}$$

$$\bar{A}(\text{kidneys}) = 1.443 \cdot 1000 \mu\text{Ci} \cdot 0.2 \cdot 1.2 \text{ hr} = 350 \mu\text{Ci-hr}$$

$$D(\text{liver}) = 1010 \mu\text{Ci-hr} \cdot 4.6 \times 10^{-5} \text{ rad}/\mu\text{Ci-hr} + 350 \mu\text{Ci-hr} \cdot 3.9 \times 10^{-6} \text{ rad}/\mu\text{Ci-hr}$$

$$D(\text{liver}) = 0.0465 \text{ rad} + 0.0014 \text{ rad} = 0.048 \text{ rad}.$$
Note that the liver contributes 97% of its total dose. Dividing by the injected activity, the dose, given these input assumptions, is 0.048 rad/mCi. So, if we redesigned the study to use 3 mCi, the liver absorbed dose would be 3 mCi × 0.048 rad/mCi = 0.14 rad.

Example
1 MBq of 99mTc is given to a patient: 40% goes to the liver and has a 10 hr biological half-time; the other 60% goes to the spleen and has an infinite biological half-time (never leaves).

\[
A_{\text{liver}} = 4 \times 10^5 \text{Bq} \times 1.443 \times \frac{6 \times 10^6 \text{s}}{6 + 10^1 \frac{\text{s}}{h}}
\]
\[
A_{\text{liver}} = 7.79 \times 10^6 \text{Bq} - s
\]
\[
A_{\text{spleen}} = 6 \times 10^5 \text{Bq} \times 1.443 \times 6h \times \frac{3600\text{s}}{h}
\]
\[
A_{\text{spleen}} = 1.87 \times 10^7 \text{Bq} - s
\]

\[
S_{\text{liver} \rightarrow \text{liver}} = 4.1 \times 10^{-5} \text{rad/µCi-hr} = 3.08 \times 10^{-6} \text{mGy/MBq} - s
\]
\[
S_{\text{spleen} \rightarrow \text{spleen}} = 3.1 \times 10^{-4} \text{rad/µCi-hr} = 2.33 \times 10^{-5} \text{mGy/MBq} - s
\]
\[
S_{\text{spleen} \rightarrow \text{liver}} = 9.6 \times 10^{-7} \text{rad/µCi-hr} = 7.2 \times 10^{-8} \text{mGy/MBq} - s
\]
\[
S_{\text{liver} \rightarrow \text{spleen}} = 9.6 \times 10^{-7} \text{rad/µCi-hr} = 7.2 \times 10^{-8} \text{mGy/MBq} - s
\]

\[
D_{\text{liver}} = 7.79 \times 10^3 \text{MBq} - s \times 3.08 \times 10^{-6} \text{mGy/MBq} - s
\]
\[
+ 1.87 \times 10^4 \text{MBq} - s \times 7.2 \times 10^{-8} \text{mGy/MBq} - s = 0.025 \text{mGy}
\]
\[
D_{\text{spleen}} = 7.79 \times 10^3 \text{MBq} - s \times 7.2 \times 10^{-8} \text{mGy/MBq} - s
\]
\[
+ 1.87 \times 10^4 \text{MBq} - s \times 2.33 \times 10^{-5} \text{mGy/MBq} - s = 0.43 \text{mGy}
\]

Example: Dose to the Fetus
MIRD Dose Estimate Report No. 1329 gives the following numbers of disintegrations for intravenous administration of 99mTc MDP.

Cortical bone 1.36 µCi-hr/µCi administered
Cancellous bone 1.36 µCi-hr/µCi administered
Kidneys 0.148 µCi-hr/µCi administered
Urinary bladder 0.782 µCi-hr/µCi administered
Remainder of body 1.64 µCi-hr/µCi administered

If 17 mCi of 99mTc-MDP has been given to a woman who is two weeks pregnant, what is the likely absorbed dose to the fetus? In early pregnancy, the dose to the nongravid uterus is a reasonably good estimate of the fetal dose, because the size and shape of the uterus relative to other organs has not changed substantially. Therefore, we can use S-values for these source organs irradiating the uterus:

\[
S_{(\text{Uterus} \leftarrow \text{Cortical bone})} = 5.7 \times 10^{-7} \text{rad/µCi-hr}
\]
\[
S_{(\text{Uterus} \leftarrow \text{Cancellous bone})} = 5.7 \times 10^{-7} \text{rad/µCi-hr}
\]
\[
S_{(\text{Uterus} \leftarrow \text{Kidneys})} = 9.4 \times 10^{-7} \text{rad/µCi-hr}
\]
\[ S(\text{Uterus} \leftarrow \text{Urinary bladder}) = 1.6 \times 10^{-5} \text{rad/\(\mu\)Ci-hr} \]
\[ S(\text{Uterus} \leftarrow \text{Total body}) = 2.6 \times 10^{-6} \text{rad/\(\mu\)Ci-hr} \]

The last S-value is not exactly what we need. It is the S-value for an organ being irradiated by activity uniformly distributed in the whole body (i.e., including bone, kidneys, etc.). The formula for calculating the S-value for remainder of the body for a given configuration of other source organs is:

\[
S(r_k \leftarrow \text{RB}) = S(r_k \leftarrow \text{TB}) \left( \frac{m_{\text{TB}}}{m_{\text{RB}}} \right) - \sum_h S(r_k \leftarrow r_h) \left( \frac{m_h}{m_{\text{RB}}} \right)
\]

where:

- \(S(r_k \leftarrow \text{RB})\) is the S-value for the remainder of the body irradiating target region \(r_k\)
- \(S(r_k \leftarrow \text{TB})\) is the S-value for the total body irradiating target region \(r_k\)
- \(S(r_k \leftarrow r_h)\) is the S-value for source region \(h\) irradiating target region \(r_k\)
- \(m_{\text{TB}}\) is the mass of the total body
- \(m_{\text{RB}}\) is the mass of the remainder of the body, that is, the total body minus all other source organs used in this problem
- \(m_h\) is the mass of source region \(h\)

For this problem, the S-value for remainder of the body to uterus is 2.7 \(\times 10^{-6}\) rad/\(\mu\)Ci-hr (4% higher than that for the total body). The total dose to the uterus is calculated as

\[
1.36 \, \mu\text{Ci-hr/\(\mu\)Ci administered} \times 5.7 \times 10^{-7} \text{rad/\(\mu\)Ci-hr/\(\mu\)Ci} = 7.8 \times 10^{-7} \text{rad/\(\mu\)Ci}
\]
\[
1.36 \, \mu\text{Ci-hr/\(\mu\)Ci administered} \times 5.7 \times 10^{-7} \text{rad/\(\mu\)Ci-hr/\(\mu\)Ci} = 7.8 \times 10^{-7} \text{rad/\(\mu\)Ci}
\]
\[
0.148 \, \mu\text{Ci-hr/\(\mu\)Ci administered} \times 9.4 \times 10^{-7} \text{rad/\(\mu\)Ci-hr/\(\mu\)Ci} = 1.4 \times 10^{-7} \text{rad/\(\mu\)Ci}
\]
\[
0.782 \, \mu\text{Ci-hr/\(\mu\)Ci administered} \times 1.6 \times 10^{-5} \text{rad/\(\mu\)Ci-hr/\(\mu\)Ci} = 1.2 \times 10^{-5} \text{rad/\(\mu\)Ci}
\]
\[
1.64 \, \mu\text{Ci-hr/\(\mu\)Ci administered} \times 2.7 \times 10^{-6} \text{rad/\(\mu\)Ci-hr/\(\mu\)Ci} = 4.4 \times 10^{-6} \text{rad/\(\mu\)Ci}
\]

TOTAL = 1.8 \times 10^{-5} \text{rad/\(\mu\)Ci}

Total dose from incident = 1.8 \times 10^{-5} \text{rad/\(\mu\)Ci} \times 17,000 \mu\text{Ci} = 0.30\text{rad.}

It would probably be more accurate to use the 57 kg model for the adult female instead of the 70 kg adult male model to calculate this estimate. Using S-values for the adult female, a dose of 2.3 \(\times 10^{-5}\) rad/\(\mu\)Ci is estimated, leading to an estimate of the total dose of 0.39 rad.

**Example: Dose to Several Organs**

In MIRD Dose Estimate Report No. 12, the following residence times are found for intravenous administration of 99mTc DTPA.

<table>
<thead>
<tr>
<th>Organ</th>
<th>Residence Time</th>
<th>S-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kidneys</td>
<td>0.092 (\mu)Ci-hr/(\mu)Ci administered</td>
<td>0.84 (\mu)Ci-hr/(\mu)Ci administered</td>
</tr>
<tr>
<td>Urinary bladder</td>
<td>(2.4 hr voiding intervals)</td>
<td>(1.72 \mu\text{Ci-hr/(\mu)Ci administered})</td>
</tr>
<tr>
<td>Remainder of body</td>
<td>(4.8 hr voiding intervals)</td>
<td>0.84 (\mu)Ci-hr/(\mu)Ci administered</td>
</tr>
</tbody>
</table>

Let’s calculate the absorbed dose to these organs and to the ovaries, testes, and red marrow. For each target organ, then, we will need all of the S-values for the three source organs. We also have two conditions to the problem: 2.4 hour and 4.8 hour voiding intervals for the urinary bladder. As in the previous example, we will have three contributions to each target organ’s total dose for
each group of cumulated activity values. An easy way to represent what proves to be a rather substantial amount of math for a simple problem is through the use of matrices. If the set of dose estimates we want is a $2 \times 6$ matrix (two sets of dose estimates by six target organs: kidneys, bladder, ovaries, testes, red marrow, and total body), this can be found by multiplication of a $2 \times 3$ matrix of cumulated activity values and a $3 \times 6$ matrix of $S$-values:

$$D = \tau S$$

$$D = \begin{bmatrix}
\bar{A}_{\text{kid}1} & \bar{A}_{\text{blad}1} & \bar{A}_{\text{RB}1} \\
\bar{A}_{\text{kid}2} & \bar{A}_{\text{blad}2} & \bar{A}_{\text{RB}2}
\end{bmatrix}$$

$$\times \begin{bmatrix}
S(\text{kid} \leftarrow \text{kid}) & S(\text{ov} \leftarrow \text{kid}) & S(\text{mar} \leftarrow \text{kid}) & S(\text{test} \leftarrow \text{kid}) & S(\text{blad} \leftarrow \text{kid}) & S(\text{TB} \leftarrow \text{kid}) \\
S(\text{kid} \leftarrow \text{blad}) & S(\text{ov} \leftarrow \text{blad}) & S(\text{mar} \leftarrow \text{blad}) & S(\text{test} \leftarrow \text{blad}) & S(\text{blad} \leftarrow \text{blad}) & S(\text{TB} \leftarrow \text{blad}) \\
S(\text{kid} \leftarrow \text{RB}) & S(\text{ov} \leftarrow \text{RB}) & S(\text{mar} \leftarrow \text{RB}) & S(\text{test} \leftarrow \text{RB}) & S(\text{blad} \leftarrow \text{RB}) & S(\text{TB} \leftarrow \text{B})
\end{bmatrix}$$

$$D = \begin{bmatrix}
0.092 & 0.842 & 2.84 \\
0.092 & 1.72 & 2.84
\end{bmatrix}$$

$$\times \begin{bmatrix}
1.9 \times 10^{-4} & 1.1 \times 10^{-6} & 3.8 \times 10^{-6} & 8.8 \times 10^{-8} & 2.8 \times 10^{-7} & 2.1 \times 10^{-6} \\
2.6 \times 10^{-7} & 7.3 \times 10^{-6} & 1.6 \times 10^{-6} & 4.7 \times 10^{-6} & 1.6 \times 10^{-4} & 1.9 \times 10^{-6} \\
1.4 \times 10^{-6} & 2.4 \times 10^{-6} & 2.9 \times 10^{-6} & 1.7 \times 10^{-6} & 1.9 \times 10^{-6} & 2.0 \times 10^{-6}
\end{bmatrix}$$

$$D = \begin{bmatrix}
D_{\text{kid}1} & D_{\text{ov}1} & D_{\text{mar}1} & D_{\text{test}1} & D_{\text{blad}1} & D_{\text{TB}1} \\
D_{\text{kid}2} & D_{\text{ov}2} & D_{\text{mar}2} & D_{\text{test}2} & D_{\text{blad}2} & D_{\text{TB}2}
\end{bmatrix}$$

$$D = \begin{bmatrix}
2.2 \times 10^{-5} & 1.3 \times 10^{-5} & 1.0 \times 10^{-5} & 8.8 \times 10^{-6} & 1.4 \times 10^{-4} & 7.5 \times 10^{-6} \\
2.2 \times 10^{-5} & 1.9 \times 10^{-5} & 1.2 \times 10^{-5} & 1.3 \times 10^{-5} & 2.8 \times 10^{-4} & 9.1 \times 10^{-6}
\end{bmatrix}$$

(units are rad/mCi)

Note from the results the increase in absorbed dose to the bladder, as well as to the gonads, from the increase in the number of disintegrations occurring in the bladder.

Endnotes


